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Entropy function for rotating extremal black holes in very special geometry

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ABSTRACT

We use the relation between extremal black hole solutions in five- and in four-dimensional $N = 2$ supergravity theories with cubic prepotentials to define the entropy function for extremal black holes with one angular momentum in five dimensions. We construct two types of solutions to the associated attractor equations.

1 Introduction

An important feature of extremal black holes in the presence of scalar fields is that these fields attain fixed values at the horizon which are determined by the black hole charges. These values are found by solving a set of so-called attractor equations, which were first given in [1, 2, 3, 4] in the context of supersymmetric black holes. The attractor equations can be obtained from a variational principle based on an entropy function [5, 6]. The value of the entropy function at the stationary point yields the macroscopic entropy of the black hole.

Extremal black holes in five dimensions can be related to extremal black holes in four dimensions. This connection is implemented by placing the five-dimensional black hole in a Taub-NUT geometry, and by using the modulus of the Taub-NUT space to interpolate between the five and the four-dimensional description. In the vicinity of the NUT charge, spacetime looks five-dimensional, whereas far away from the NUT the spacetime looks four-dimensional. This connection was first established in [7, 8] for supersymmetric black holes in the context of $N = 2$ supergravity theories that in four dimensions are based on cubic prepotentials, and was further discussed in [9].

In the following, we focus on rotating extremal black holes in five dimensions which are connected to static extremal black holes in four dimensions in the way described above. We use this link to define the entropy function for these rotating black hole solutions in the context of $N = 2$ supergravity theories with cubic prepotentials. In four dimensions, the static extremal black holes we consider carry charges (P^I, Q_I), where $P^0 \neq 0$ corresponds to the NUT charge in five dimensions. These four-dimensional black holes are connected to rotating five-dimensional black holes with one independent angular momentum parameter. The five-dimensional $N = 2$ supergravity theories contain Chern-Simons terms for the abelian gauge fields, so that the definition of the entropy function given in [5, 6] cannot be directly applied whenever these terms play a role for the given background. Therefore, we define the entropy function for these rotating five-dimensional black holes to equal the entropy function of the associated static black holes in four dimensions. The latter was computed for $N = 2$ supergravity theories in [10, 11]. Then, we specialize to the case of black holes with non-vanishing charges (P^0, Q_I), which in five dimensions correspond to rotating electrically charged extremal black holes in a Taub-NUT geometry. Extremization of the entropy function yields a set of attractor equations for the various parameters characterizing the near-horizon solution. We check that these attractor equations are equivalent to the equations of motion in five dimensions evaluated in the black hole background. We construct two types of solutions to the attractor equations and we compute their entropy.

Our approach for defining the entropy function in the presence of Chern-Simons terms is based on dimensional reduction, and is therefore similar to the approach used in [12] for defining the entropy function of the three-dimensional BTZ black hole. Related results for rotating AdS_5 black holes have appeared in [13].

2 Extremal black holes in five and four dimensions

Extremal black holes in five dimensions can be connected to extremal black holes in four dimensions, as described in the introduction. In the following, we focus on rotating black holes in five dimensions which are connected to static black holes in four dimensions. The associated near-horizon geometries are related by dimensional reduction over a compact direction of radius R. In the context of five-dimensional theories based on n abelian gauge fields A_5^A and real scalar fields X^A ($A = 1, \dots, n$) coupled to gravity, the reduction is based on the following standard formulae (see for instance [14]),

$$\begin{aligned} ds_5^2 &= \mathrm{e}^{2\phi}\, ds_4^2 + \mathrm{e}^{-4\phi}\, (dx^5 - A_4^0)^2 \quad , \quad dx^5 = R\, d\psi \;, \\ A_5^A &= A_4^A + C^A\, (dx^5 - A_4^0) \;, \\ \hat{X}^A &= \mathrm{e}^{-2\phi}\, X^A \;, \end{aligned} \tag{2.1}$$

where the A_4^I denote the four-dimensional abelian gauge fields (with $I = 0, A$).

We will focus on $N = 2$ supergravity theories that are based on cubic prepotentials in four dimensions. As we review in appendix A, the rescaled scalar fields $\hat{X}^A$ and the Kaluza-Klein scalars C^A are combined into the four-dimensional complex scalar fields z^A [14],

$$z^A = C^A + \mathrm{i}\hat{X}^A \;. \tag{2.2}$$

We take the fields C^A and $\hat{X}^A$, and hence also z^A, to be dimensionless.

The near-horizon geometry of the rotating five-dimensional black hole is taken to be a squashed $AdS_2 \times S^3$ given by [13]

$$ds_5^2 = v_1(-r^2dt^2 + \frac{dr^2}{r^2}) + \frac{v_2}{4}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + \frac{v_2 v_3}{4}\,(d\psi + \cos\theta\, d\phi - \alpha\, r dt)^2 \;, \tag{2.3}$$

where $\theta \in [0, \pi]$, $\phi \in [0, 2\pi)$, $\psi \in [0, 4\pi)$. The parameters v_1, v_2, v_3 and α are constant. The near-horizon geometry of the associated static four-dimensional black hole is of the $AdS_2 \times S^2$ type,

$$ds_4^2 = \tilde{v}_1(-r^2dt^2 + \frac{dr^2}{r^2}) + \tilde{v}_2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \tag{2.4}$$

with constant parameters $\tilde{v}_1$ and $\tilde{v}_2$. Using (2.1), we find the following relations,

$$\begin{aligned} \mathrm{e}^{-4\phi} &= \frac{v_2 v_3}{4R^2} \;, \\ A_4^0 &= R\,(-\cos\theta d\phi + \alpha r dt) \;, \end{aligned} \tag{2.5}$$

as well as

$$\tilde{v}_1 = v_1\sqrt{\frac{v_2 v_3}{4R^2}} \quad , \quad \tilde{v}_2 = \frac{v_2}{4}\sqrt{\frac{v_2 v_3}{4R^2}} \;, \tag{2.6}$$

and hence

$$\tilde{v}_1\, \tilde{v}_2 = \frac{v_1\, v_2^2\, v_3}{16R^2} \quad , \quad \frac{\tilde{v}_1}{\tilde{v}_2} = 4\,\frac{v_1}{v_2} \;. \tag{2.7}$$

We denote the electric fields in four and five dimensions by $F_{rt} = e$. Hence, we rewrite A_4^0 as

$$A_4^0 = e_4^0\, r\, dt - p^0 R \cos\theta d\phi \;, \tag{2.8}$$

with $e_4^0 = \alpha R$ and the NUT charge $p^0 = 1$.

3 Entropy function for rotating extremal black holes in five dimensions

The entropy function of [5, 6] is derived from the reduced Lagrangian. The reduced Lagrangian $\mathcal{F}$ is obtained by evaluating the Lagrangian in the near-horizon black hole background and integrating over the horizon. In five and four dimensions,

$$\begin{aligned}
\mathcal{F}_5 &= \int d\psi\, d\theta\, d\phi \sqrt{-G}\, \mathcal{L}_5 \ , \\
\mathcal{F}_4 &= \int d\theta\, d\phi \sqrt{-g}\, \mathcal{L}_4 \ .
\end{aligned} \tag{3.1}$$

In the presence of Chern-Simons terms, however, the definition of the entropy function given in [5, 6] is not directly applicable whenever these terms play a role for the given background. This is the situation encountered in $N = 2$ supergravity theories in five dimensions, but not in four dimensions. Therefore, we use dimensional reduction to define the entropy function $\mathcal{E}_5$ for rotating black holes in five dimensions in terms of the entropy function $\mathcal{E}_4$ for the associated four-dimensional black holes,

$$\mathcal{E}_5 = \mathcal{E}_4 \ . \tag{3.2}$$

3.1 Rotating electrically charged black holes in five dimensions

Here we consider rotating electrically charged extremal black hole solutions in $N = 2$ supergravity theories in five dimensions. The bosonic part of the five-dimensional Lagrangian is given by (A.6). The black hole solutions carry NUT charge $p^0 = 1$ as well as electric charges q_A. The near-horizon solution is specified in terms of constant scalars X^A, the line element (2.3) and the five-dimensional gauge potentials A_5^A,

$$A_5^A = e_5^A\, r\, dt + C^A\, R\, \left(d\psi + \cos\theta d\phi\right) \ , \tag{3.3}$$

where $F_{rt}^A = e_5^A$ denotes the electric field in five dimensions. Both e_5^A and C^A are constant.

These five-dimensional rotating extremal black holes are connected to static electrically charged extremal black holes in four dimensions with constant scalars z^A, line element (2.4) and four-dimensional gauge potentials A_4^I given by (2.8) and

$$A_4^A = e_4^A\, r\, dt \ . \tag{3.4}$$

The five- and four-dimensional electric fields are related by

$$e_5^A = e_4^A - C^A\, e_4^0 = e_4^A - \alpha\, R\, C^A \tag{3.5}$$

according to (2.1). In our conventions, the electric fields in five and four dimensions have length dimension one.

As reviewed in appendix A, the five- and four-dimensional actions (A.6) and (A.10) are identical upon dimensional reduction over x^5, up to boundary terms which are usually discarded and which arise when integrating the Chern-Simons term in (A.6) by parts. However, when evaluating these actions in a background with constant C^A, as is the case for the near-horizon solutions under consideration, they are not any longer equal to one another. Namely, evaluating the Chern-Simons term in (A.6) for constant C^A, and using $F_5^A = F_4^A - C^A\, F_4^0$ (see (2.1)), we obtain with the help of (A.18)

$$\begin{aligned} C_{ABC}\, F_5^A \wedge F_5^B \wedge A_5^C \;&=\; \frac{1}{2}\, R\, d\psi\, d^4x\, \sqrt{-g}\, \mathrm{Re}\mathcal{N}_{IJ}\, F_4^I\, \tilde{F}_4^J \\ &\quad - R\, C_{ABC}\, \left(C^A\, C^B\, F_4^C - \frac{2}{3}\, C^A\, C^B\, C^C\, F_4^0 \right) \wedge F_4^0 \wedge d\psi\ . \end{aligned} \tag{3.6}$$

Thus, the actions differ by

$$\begin{aligned} 8\pi\, (S_5 - S_4) \;&=\; \frac{1}{6G_4} \int d^4x\, \sqrt{-g}\, \mathrm{Re}\mathcal{N}_{IJ}\, F_4^I\, \tilde{F}_4^J \\ &\quad + \frac{1}{6G_4} \int C_{ABC}\, \left(C^A\, C^B\, F_4^C - \frac{2}{3}\, C^A\, C^B\, C^C\, F_4^0 \right) \wedge F_4^0\ . \end{aligned} \tag{3.7}$$

Similarly, in the background specified by (3.3), the reduced Lagrangians (3.1) differ by

$$\mathcal{F}_5 - \mathcal{F}_4 = \frac{1}{12 G_4}\, R\, C_{ABC}\, C^A\, C^B\, e_4^C\ . \tag{3.8}$$

This has to be taken into account when using (3.2) to define the entropy function in five dimensions in terms of $\mathcal{E}_4$. The entropy function of static extremal black holes in four dimensions is the Legendre transform of the reduced Lagrangian $\mathcal{F}_4$ with respect to the electric fields and reads [5]

$$\mathcal{E}_4 = 2\pi \left(-\frac{1}{2}\, e_4^I\, Q_I\, G_4^{-1/2} - \mathcal{F}_4 \right)\ , \tag{3.9}$$

where we denote the four-dimensional electric charges by Q_I. The normalizations are as in [10, 11], with the additional $G_4^{-1/2}$ to ensure that $\mathcal{E}_4$ is dimensionless. Using (3.5), (3.8) and (3.2), we now express (3.9) as

$$\begin{aligned} \mathcal{E}_5 \;&=\; 2\pi \left[-\frac{1}{2}\, \alpha\, \left(J + R\, C^A \left(q_A\, G_5^{-1/3} - \frac{2\pi\, R^2}{3 G_5}\, C_{ABC}\, C^B\, C^C \right) \right) \right. \\ &\quad \left. - \frac{1}{2}\, e_5^A \left(q_A\, G_5^{-1/3} - \frac{2\pi\, R^2}{3G_5}\, C_{ABC}\, C^B\, C^C \right) - \mathcal{F}_5 \right]\ , \end{aligned} \tag{3.10}$$

where the five-dimensional quantities (J, q_A) are given in terms of the four-dimensional electric charges (Q_0, Q_A) by

$$\begin{aligned} J \;&=\; Q_0\, R\, G_4^{-1/2}\ , \\ q_A\, G_5^{-1/3} \;&=\; Q_A\, G_4^{-1/2}\ . \end{aligned} \tag{3.11}$$

In (3.18) below, J will be related to the angular momentum in five-dimensions. Observe that in the presence of the C^A, the electric charges q_A are shifted by a term proportional to $C_{ABC}\, C^B\, C^C$. This shift, which is due to (3.8) and thus has its origin in the presence of the Chern-Simons term in the five-dimensional action (A.6), has also been observed in [13]. In addition, we note that J also gets shifted by terms involving C^A. This shift ensures that extrema of $\mathcal{E}_5$ satisfy all the five-dimensional equations of motion. This we now demonstrate by explicitly checking the equation of motion for $A^A_{5\psi}$, as follows. Using (3.3), we compute

$$\begin{aligned}\mathcal{F}_5 &= \pi\,\frac{v_1\,(v_2^3\,v_3)^{1/2}}{4G_5}\left[-\frac{1}{v_1}+\frac{4-v_3}{v_2}+\frac{v_2\,v_3\,\alpha^2}{16v_1^2}+\frac{G_{AB}\,e_5^A\,e_5^B}{2v_1^2}-8R^2\,\frac{G_{AB}\,C^A\,C^B}{v_2^2}\right]\\ &\quad -\frac{2\pi}{3G_5}\,R^2\,C_{ABC}\,C^A\,C^B\,e_5^C\ . \end{aligned} \tag{3.12}$$

Then, varying the entropy function $\mathcal{E}_5$ with respect to the electric fields e_5^A and setting $\partial_e\mathcal{E}_5=0$ yields

$$\frac{\pi}{4G_5}\,\frac{(v_2^3\,v_3)^{1/2}}{v_1}\,G_{AB}\,e_5^B=-\frac{1}{2}\,\hat{q}_A\ , \tag{3.13}$$

while varying with respect to C^A and setting $\partial_C\mathcal{E}_5=0$ gives

$$-\frac{\alpha}{2}\,\hat{q}_A+\frac{2\pi R}{G_5}\,C_{ABC}\,C^B\,e_5^C+\frac{4\pi R}{G_5}\,\frac{v_1(v_2^3\,v_3)^{1/2}}{v_2^2}\,G_{AB}C^B=0\ , \tag{3.14}$$

where we introduced

$$\hat{q}_A=q_A\,G_5^{-1/3}-\frac{2\pi R^2}{G_5}C_{ABC}\,C^B\,C^C\ , \tag{3.15}$$

for convenience. Combining (3.13) and (3.14) results in

$$\alpha\,\frac{(v_2^3\,v_3)^{1/2}}{v_1}\,G_{AB}\,e_5^B+8R\,C_{ABC}\,C^B\,e_5^C+16R\,\frac{v_1(v_2^3\,v_3)^{1/2}}{v_2^2}\,G_{AB}C^B=0\ , \tag{3.16}$$

which is precisely the equation of motion for $A^A_{5\psi}$ evaluated in the black hole background. Observe that when $\alpha\neq 0$, then also $C^A\neq 0$. We also note that when expressed in terms of four-dimensional quantities, $\hat{q}_A$ equals $\hat{q}_A=G_4^{-1/2}\left(Q_A-\mathrm{Re}\mathcal{N}_{A0}\,P^0\right)$, where P^0 is given by (3.30).

The entropy function (3.10) depends on a set of constant parameters, namely e_5^A, X^A, C^A, v_1, v_2, v_3 and α, whose horizon values are determined by extremizing $\mathcal{E}_5$. To this end, we compute the (remaining) extremization equations. Inserting (3.13) into (3.10) gives

$$\begin{aligned}\mathcal{E}_5 &= 2\pi\,\alpha\left[-\frac{1}{2}J-\frac{1}{2}R\,C^A\left(q_A\,G_5^{-1/3}-\frac{2\pi R^2}{3G_5}\,C_{ABC}\,C^B\,C^C\right)\right]+G_5\,\frac{v_1}{(v_2^3\,v_3)^{1/2}}\,\hat{q}_A\,G^{AB}\,\hat{q}_B\\ &\quad -\frac{\pi^2}{2G_5}v_1(v_2^3\,v_3)^{1/2}\left[-\frac{1}{v_1}+\frac{4-v_3}{v_2}+\frac{v_2\,v_3\alpha^2}{16v_1^2}-8R^2\frac{G_{AB}C^AC^B}{v_2^2}\right]\ . \end{aligned} \tag{3.17}$$

Demanding $\partial_\alpha\mathcal{E}_5=0$ results in the expression for the angular momentum,

$$\frac{\pi}{32G_5}\,\frac{v_2^{5/2}v_3^{3/2}}{v_1}\,\alpha=-\frac{1}{2}J-\frac{1}{2}R\,C^A\left(q_A\,G_5^{-1/3}-\frac{2\pi R^2}{3G_5}\,C_{ABC}\,C^B\,C^C\right)\ . \tag{3.18}$$

Computing $\partial_{v_i}\mathcal{E}_5=0$ (with $i=1,2,3$), we obtain

$$\begin{aligned} v_1 &= \frac{v_2}{4}\ ,\\ v_2\,v_3\left[2v_2+v_2\,v_3(1-2\alpha^2)\right] &= \frac{2G_5^2}{\pi^2}\,\hat{q}_A\,G^{AB}\,\hat{q}_B\ ,\\ 2v_2-v_2v_3(2-\alpha^2) &= 8R^2\,G_{AB}\,C^A\,C^B\ . \end{aligned} \tag{3.19}$$

Observe that the first of these conditions yields $\tilde{v}_1 = \tilde{v}_2$, as can be seen from (2.7). This implies the vanishing of the Ricci scalar for the associated four-dimensional geometry.

Inserting the relations (3.18) and (3.19) into (3.17) results in

$$\mathcal{E}_5 = \frac{\pi^2}{2G_5} \left(v_2^3\, v_3\right)^{1/2} \ , \tag{3.20}$$

which exactly equals the macroscopic entropy $\mathcal{S}_{\rm macro} = A_5/(4G_5)$ of the rotating black hole, where A_5 denotes the horizon area.

Introducing the abreviations

$$\begin{aligned} \Omega &= \frac{2G_5^2}{\pi^2} \frac{1}{\sqrt{v_2\, v_3}}\, \hat{q}_A\, G^{AB}\, \hat{q}_B \ , \\ \Delta &= 8R^2 \sqrt{v_2\, v_3}\, G_{AB}\, C^A\, C^B \ , \\ \Gamma &= \frac{8G_5}{\pi} \left[-\frac{1}{2} J - \frac{1}{2} R\, C^A \left(q_A\, G_5^{-1/3} - \frac{2\pi R^2}{3G_5}\, C_{ABC}\, C^B\, C^C \right) \right] \ , \end{aligned} \tag{3.21}$$

we obtain from (3.18) and (3.19) the following two equations,

$$\begin{aligned} 3(v_2\, v_3)^{3/2} - 3\frac{\Gamma^2}{(v_2 v_3)^{3/2}} &= \Omega - \Delta \ , \\ \sqrt{v_2\, v_3}\, (6v_2 - 3v_2 v_3) &= \Omega + 2\Delta \ . \end{aligned} \tag{3.22}$$

Solving the first of these equations yields (with $v_2 v_3$ positive)

$$(v_2\, v_3)^{3/2} = \frac{1}{6} (\Omega - \Delta) + \sqrt{\Gamma^2 + \frac{1}{36} (\Omega - \Delta)^2} \ . \tag{3.23}$$

Inserting this into the second equation of (3.22) gives

$$\left(v_2^3\, v_3\right)^{1/2} = \frac{1}{4} (\Omega + \Delta) + \frac{1}{2} \sqrt{\Gamma^2 + \frac{1}{36} (\Omega - \Delta)^2} \ . \tag{3.24}$$

Thus, by taking suitable ratios of (3.23) and (3.24), we obtain v_2 and v_3 expressed in terms of Ω, Δ and Γ. Now, recalling the definition of $\hat{X}^A$ in (2.1) and using (2.5), we have $\sqrt{v_2\, v_3}\, G_{AB} = 2R\, \hat{G}_{AB}$, where

$$\hat{G}_{AB} = -C_{ABC}\, \hat{X}^C + 9\, \frac{\hat{X}_A\, \hat{X}_B}{\hat{V}} \ , \tag{3.25}$$

with $\hat{X}_A$ and $\hat{V}$ defined in (A.19). Therefore Ω, Δ and Γ, and hence also the horizon area (3.24), are entirely determined in terms of the scalar fields $\hat{X}^A$ and C^A and the charges. The horizon values of $\hat{X}^A$ and C^A are in turn determined in terms of the charges by solving the respective extremization equations. The extremization equations for the C^A are given by (3.14), while the extremization equations for the $\hat{X}^A$ are obtained by setting $\partial_{X^A} \mathcal{E}_5 = 0$. Rather than computing the horizon values in this way, we will determine them by solving the associated attractor equations in four dimensions. This will be done in the next subsection.

Finally, let us consider static black holes. When the rotation parameter α is set to zero, we have $\Gamma = 0$ and (3.16) can be abbreviated as $D_{AB}\, C^B = 0$. In the following we will assume that D_{AB} is invertible so that $C^A = 0$. We then infer from (3.19) and (3.21) that $v_3 = 1$, $\Delta = 0$ and

$$\Omega = \frac{G_5^{4/3}}{\pi^2 R} \, q_A \, \hat{G}^{AB} \, q_B \ , \tag{3.26}$$

which is the black hole potential in five dimensions for static electrically charged black holes [15]. Using (3.11), we obtain for (3.20),

$$\mathcal{E}_5 = \frac{2\pi}{3} \, Q_A \, \hat{G}^{AB} \, Q_B \ . \tag{3.27}$$

From (A.18) and (3.25) we infer that $\hat{G}_{AB} = -\mathrm{Im}\mathcal{N}_{AB}$. With the help of (A.18), (A.19) and (3.24) we compute

$$\mathrm{Im}\mathcal{N}_{00} = -\hat{V} = \frac{1}{12\pi} \frac{G_5}{R^3} \, Q_A \, [(\mathrm{Im}\mathcal{N})^{-1}]^{AB} \, Q_B \ , \tag{3.28}$$

where we used $\mathrm{Im}\mathcal{N}_{A0} = 0$. It follows that we can rewrite (3.27) as

$$\mathcal{E}_5 = -\frac{2\pi}{4} \left[(P^0)^2 \, \mathrm{Im}\mathcal{N}_{00} + Q_A \, [(\mathrm{Im}\mathcal{N})^{-1}]^{AB} \, Q_B \right] \ , \tag{3.29}$$

where

$$P^0 = p^0 \, \frac{R}{G_4^{1/2}} \quad , \quad p^0 = 1 \ . \tag{3.30}$$

Thus, (3.27) precisely equals the four-dimensional black hole potential [16, 17, 18],

$$\mathcal{E}_4 = -\frac{2\pi}{4} (Q_I - \mathcal{N}_{IK} P^K) \, [(\mathrm{Im}\mathcal{N})^{-1}]^{IJ} \, (Q_J - \bar{\mathcal{N}}_{JL} P^L) \ , \tag{3.31}$$

for the case at hand with $C^A = 0$ and non-vanishing charges (P^0, Q_A), as it should. In (3.31) (P^I, Q_J) denote the magnetic and electric charges in four dimensions, respectively.

3.2 Attractor equations and examples

The four-dimensional entropy function (3.31) can be rewritten into [11]

$$\mathcal{E}_4 = \pi \left[\Sigma + \left(\mathcal{Q}_I - F_{IJ} \mathcal{P}^J \right) N^{IK} \left(\mathcal{Q}_K - \bar{F}_{KL} \mathcal{P}^L \right) \right] \ , \tag{3.32}$$

where

$$\begin{aligned}
\Sigma &= -\mathrm{i} \left(\bar{Y}^I F_I - Y^I \bar{F}_I \right) - Q_I (Y^I + \bar{Y}^I) + P^I (F_I + \bar{F}_I) \ , \\
N_{IJ} &= \mathrm{i} \left(\bar{F}_{IJ} - F_{IJ} \right) \ , \\
\mathcal{Q}_I &= Q_I + \mathrm{i} (F_I - \bar{F}_I) \ , \\
\mathcal{P}^I &= P^I + \mathrm{i} (Y^I - \bar{Y}^I) \ .
\end{aligned} \tag{3.33}$$

For the notation cf. appendix A. The scalar fields (2.2) are expressed in terms of the Y^I by $z^A = Y^A / Y^0$. The horizon values of the scalar fields $\hat{X}^A$ and C^A can be conveniently determined by solving the attractor equations for the Y^I in four dimensions, which read [19, 11]

$$-2(\mathcal{Q}_J - F_{JK} \, \mathcal{P}^K) + \mathrm{i} (\mathcal{Q}_I - \bar{F}_{IM} \, \mathcal{P}^M) N^{IR} \, F_{RSJ} \, N^{SK} (\mathcal{Q}_K - \bar{F}_{KL} \, \mathcal{P}^L) = 0 \ . \tag{3.34}$$

Contracting with Y^I results in

$$\mathrm{i}\left(\bar{Y}^I F_I - Y^I \bar{F}_I\right) = P^I F_I - Q_I Y^I . \tag{3.35}$$

Supersymmetric black holes satisfy $\mathcal{Q}_I = \mathcal{P}^J = 0$.

In the following, we will discuss two classes of four-dimensional non-supersymmetric extremal black holes which are connected to five-dimensional black holes. These have a non-vanishing P^0 given by (3.30). The first class consists of black holes with non-vanishing charges (P^0, Q_A) in heterotic-like theories with prepotential $F(Y) = -Y^1 Y^a \eta_{ab} Y^b / Y^0$, where η_{ab} denotes a symmetric matrix with the inverse η^{ab} ($\eta^{ab}\eta_{bc} = \delta^a_c$) and $a, b = 2, \ldots, n$. These black holes are static in five dimensions. Taking $P^0 > 0$ and $Q_1 \, Q_a \, \eta^{ab} \, Q_b < 0$, we find that the attractor equations (3.34) are solved by

$$\begin{aligned} Y^0 &= -\frac{\mathrm{i}}{4}\, P^0 , \\ Y^1 &= \frac{1}{8}\sqrt{-\frac{P^0\, Q_a\, \eta^{ab}\, Q_b}{Q_1}} , \\ Y^a &= -\frac{1}{4}\sqrt{-\frac{P^0\, Q_1}{Q_a\, \eta^{ab}\, Q_b}}\; \eta^{ab}\, Q_b . \end{aligned} \tag{3.36}$$

The z^A read,

$$\begin{aligned} z^1 &= \mathrm{i}\,\hat{X}^1 = \frac{\mathrm{i}}{2}\sqrt{-\frac{Q_a\, \eta^{ab}\, Q_b}{P^0\, Q_1}} , \\ z^a &= \mathrm{i}\,\hat{X}^a = -\mathrm{i}\sqrt{-\frac{Q_1}{P^0\, Q_a\, \eta^{ab}\, Q_b}}\; \eta^{ab}\, Q_b . \end{aligned} \tag{3.37}$$

Requiring $\hat{V} > 0$ for consistency (see (A.19)) restricts the charges to $Q_a\, \eta^{ab}\, Q_b > 0$ and $Q_1 < 0$. Using (3.36), (3.32), (3.30) and (3.11), the entropy is computed to be

$$\mathcal{E}_5 = \pi\sqrt{-P^0\, Q_1\, Q_a\, \eta^{ab}\, Q_b} = \frac{\sqrt{\pi}}{2}\,\sqrt{-q_1\, q_a\, \eta^{ab}\, q_b} . \tag{3.38}$$

Upon performing the rescaling $q_A \to (4\pi)^{1/3}\, q_A$, the entropy (3.38) attains its standard form. For the case $n = 3$ with non-vanishing $\eta_{23} = \eta_{32} = \frac{1}{2}$, the so-called STU model, the above solution has been given in [20] and found to be stable. Requiring the moduli S, T and U to lie in the Kähler cone imposes the additional restriction $Q_2 < 0$ and $Q_3 < 0$.

The solution (3.36) is non-supersymmetric in four dimensions, since $\mathcal{Q}_A \neq 0, \mathcal{P}^0 \neq 0$. We now check the supersymmetry of the associated five-dimensional solution. An electrically charged supersymmetric solution in five dimensions satisfies the condition $\mathcal{A}_A = 0$ [21, 22, 23, 24], where in our conventions (see appendix A)

$$\mathcal{A}_A = q_A - 2\,\mathrm{e}^{6\phi}\, Z(\hat{X})\, \hat{X}_A \quad , \quad Z(\hat{X}) = q_A\, \hat{X}^A , \tag{3.39}$$

with $\hat{X}_A$ given in (A.19). Computing $\mathcal{A}_A$ for the solution (3.36) using (3.11), we find that $\mathcal{A}_A = G_5^{1/3}\, G_4^{-1/2}\, (Q_A + 3P^0\, \hat{X}_A) = 0$. The entropy (3.38) takes the supersymmetric form

$\mathcal{E}_5 = (2\pi)^{1/2}\, 3^{-3/2}\, |q_A\, X^A|^{3/2}$. Solutions which are supersymmetric from a higher-dimensional point of view, but non-supersymmetric from a lower-dimensional point of view, have been discussed in [25, 26] and occur when dimensionally reducing geometries that are $U(1)$-fibrations, as in our case.

The second class consists of black holes with non-vanishing charges (P^0, Q_0). They correspond to rotating black holes in five dimensions, of the type discussed in [27, 28, 29, 6, 30, 31, 32], which are not supersymmetric. We consider the prepotential $F(Y) = -Y^1\, Y^2\, Y^3/Y^0$. Taking $P^0\, Q_0 > 0$ and $\mathrm{Re} z^A = 0$, we find that the attractor equations (3.34) are solved by

$$\begin{aligned} Y^0 &= -\frac{(1-\mathrm{i})}{8}\, P^0 \,, \\ Y^1\, Y^2\, Y^3 &= \mathrm{i}(1-\mathrm{i})^3\, \frac{(P^0)^2\, Q_0}{512} \,. \end{aligned} \tag{3.40}$$

Observe that the attractor equations do not determine the individual values Y^1, Y^2 and Y^3, because the entropy function has two flat directions. The coupling constant $\mathrm{Im}\mathcal{N}_{00}$ and the entropy are, however, determined in terms of the charges. The former takes the value $-\mathrm{Im}\mathcal{N}_{00} = \mathrm{i} z^1\, z^2\, z^3 = Q_0/P^0 > 0$. We also find that $\hat{V} > 0$, as required by consistency. From (3.23) and (3.24) we obtain $v_2 = \frac{1}{2}\, |\Gamma|^{2/3}$ and $v_3 = 2$. Using (3.40), (3.32), (3.30) and (3.11), the entropy is computed to be

$$\mathcal{E}_5 = \pi\, P^0\, Q_0 = \pi\, J \,. \tag{3.41}$$

We close this section by displaying the relation between the five-dimensional quantity $Z(\hat{X})\, \mathrm{e}^{6\phi}$ appearing in (3.39) and the four-dimensional Y^0 for the case of static black holes with $C^A = 0$. From (3.35) we obtain (with $Q_0 = P^A = 0$, and with P^0 given by (3.30))

$$Z(\hat{X})\, \mathrm{e}^{6\phi} = \frac{\mathrm{i}}{2}\, \frac{G_5^{1/3}}{G_4^{1/2}} \left(-8\, Y^0 + \mathrm{i} \frac{R}{\sqrt{G_4}} \right) \,, \tag{3.42}$$

where we used $Y^0 = -\bar{Y}^0$, which follows from the reality of (3.42). For a supersymmetric solution in four dimensions, $\mathcal{P}^0 = 0$ and hence $Y^0 = \mathrm{i} P^0/2$, so that

$$Z(\hat{X})\, \mathrm{e}^{6\phi} = 6\pi\, R^2\, G_5^{-2/3} \,. \tag{3.43}$$

4 Conclusions

In the context of $N = 2$ supergravity theories with cubic prepotentials, we used the relation between extremal black hole solutions in four and in five dimensions to define the entropy function for rotating extremal black holes in terms of the entropy function for static black holes in four dimensions. We focused on rotating electrically charged black holes with one independent angular momentum parameter, for simplicity, and we discussed two classes of solutions. General charged static black holes in four dimensions also carry magnetic charges P^A, and these charges can be easily incorporated into the discussion given above by adding a term $-P^A\, R\, \cos\theta\, d\phi$ to both (3.3) and (3.4). Their entropy function is given by (3.32), and the entropy function of the associated five-dimensional rotating black holes is then defined by (3.2).

In five dimensions, rotating extremal black holes may carry two independent angular momentum parameters [33]. These black holes will be connected to rotating extremal black holes in four dimensions. The entropy function of these five-dimensional black holes can then again be defined in terms of the entropy function of the associated rotating four-dimensional black holes. The entropy function for rotating attractors in four dimensions has recently been discussed in [6].

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A $N = 2$ supergravity actions and dimensional reduction

Here we review various elements of $N = 2$ supergravity theories in four and in five dimensions. We also review the reduction of the five-dimensional action based on very special geometry to the four-dimensional action based on special geometry. This will explain our conventions, which differ slightly from the ones used in [14, 34, 35, 9]. For notational simplicity, we drop the subscripts on the five- and four-dimensional gauge fields.

The five-dimensional $N = 2$ supergravity action is based on the cubic polynomial [14]

$$V = \frac{1}{6} C_{ABC} X^A X^B X^C \,, \tag{A.1}$$

where the X^A are real scalar fields satisfying the constraint $V = \text{constant}$. The five-dimensional gauge couplings $G_{AB}(X)$ are given by

$$G_{AB}(X) = -\frac{1}{2} \partial_A \partial_B \log V|_{V=\text{constant}} \,, \tag{A.2}$$

and hence,

$$G_{AB}(X) = V^{-1} \left(-\frac{1}{2} C_{ABC} X^C + \frac{9}{2} \frac{X_A X_B}{V} \right) \,, \tag{A.3}$$

where we defined

$$X_A = \frac{1}{6} C_{ABC} X^B X^C \,. \tag{A.4}$$

Observe that

$$G_{AB}\, X^A X^B = \frac{3}{2} \quad , \quad X^A \partial_i X_A = 0 \,. \tag{A.5}$$

Here $\partial_i X^A = \frac{\partial}{\partial \varphi^i} X^A(\varphi)$, where φ^i denote the physical scalar fields with target space metric $g_{ij} = G_{AB}\, \partial_i X^A\, \partial_j X^B$.

The bosonic part of the five-dimensional $N=2$ supergravity action reads

$$
\begin{aligned}
S_5 \;=\; & \frac{1}{8\pi G_5}\left[\int d^5x\sqrt{-G}\left(\frac{1}{2}R_G-\frac{1}{2}\,G_{AB}\,\partial_M X^A\,\partial^M X^B-\frac{1}{4}G_{AB}\,F^A_{MN}F^{BMN}\right)\right.\\
& \left.-\frac{1}{6}\int C_{ABC}\,F^A\wedge F^B\wedge A^C\right] , \qquad (A.6)
\end{aligned}
$$

where G denotes the determinant of the spacetime metric in five dimensions.

The four-dimensional $N=2$ supergravity action, on the other hand, is based on the prepotential [36, 37]

$$
F(Y)=-\frac{1}{6}\,\frac{C_{ABC}\,Y^AY^BY^C}{Y^0}\;, \qquad (A.7)
$$

where the Y^I are complex scalar fields ($I=0,A$). The four-dimensional gauge couplings $\mathcal{N}_{IJ}$ are given by

$$
\mathcal{N}_{IJ}=\bar{F}_{IJ}+2\mathrm{i}\frac{\mathrm{Im}\,F_{IK}\;\mathrm{Im}\,F_{JL}\,Y^KY^L}{\mathrm{Im}\,F_{MN}\,Y^MY^N}\;, \qquad (A.8)
$$

where $F_I=\partial F/\partial Y^I$, $F_{IJ}=\partial^2 F/\partial Y^I\partial Y^J$. The four-dimensional physical scalar fields z^A are

$$
z^A=\frac{Y^A}{Y^0}\;. \qquad (A.9)
$$

The bosonic part of the four-dimensional $N=2$ supergravity action reads

$$
\begin{aligned}
S_4 \;=\; & \frac{1}{8\pi G_4}\int d^4x\,\sqrt{-g}\,\left(\frac{1}{2}R_g-g_{A\bar{B}}\,\partial_\mu z^A\partial^\mu\bar{z}^B+\frac{1}{4}\,\mathrm{Im}\mathcal{N}_{IJ}\,F^I_{\mu\nu}\,F^{J\mu\nu}\right.\\
& \left.-\frac{1}{4}\mathrm{Re}\mathcal{N}_{IJ}\,F^I_{\mu\nu}\,\tilde{F}^{J\mu\nu}\right) , \qquad (A.10)
\end{aligned}
$$

where g denotes the determinant of the spacetime metric in four dimensions, and where $\tilde{F}^{Jab}=\frac{1}{2}\varepsilon_{abcd}F^{Jcd}$ with $\varepsilon_{0123}=1$. The quantity $g_{A\bar{B}}$ is the Kähler metric $g_{A\bar{B}}=\frac{\partial}{\partial z^A}\frac{\partial}{\partial\bar{z}^B}K$ computed from the Kähler potential $K(z,\bar{z})$. For the prepotential (A.7), the Kähler potential reads

$$
\mathrm{e}^{-K(z,\bar{z})}=\frac{\mathrm{i}}{6}C_{ABC}(z^A-\bar{z}^A)(z^B-\bar{z}^B)(z^C-\bar{z}^C)\;. \qquad (A.11)
$$

Now we perform the reduction of (A.6) along x^5 down to four dimensions using (2.1). We take the various fields to be independent of the fifth coordinate x^5. Setting $x^5=R\psi\,,\;0\leq\psi<4\pi$, we use that the five- and four-dimensional Newton constants are related by

$$
G_5=4\pi\,R\,G_4\;. \qquad (A.12)
$$

Reducing the gauge kinetic terms $G_{AB}F^AF^B$ gives rise to a scalar kinetic term of the form

$$
-\frac{1}{4}\sqrt{-G}\,G_{AB}\,F^AF^B\to-\frac{1}{2}\,\sqrt{-g}\,\mathrm{e}^{4\phi}\,G_{AB}\,\partial_\mu C^A\partial^\mu C^B\;, \qquad (A.13)
$$

whereas reducing $R_G-G_{AB}\partial_M X^A\partial^M X^B$ gives rise to scalar kinetic terms for $\hat{X}^A=\mathrm{e}^{-2\phi}\,X^A$,

$$
\sqrt{-G}\left(\frac{1}{2}R_G-\frac{1}{2}G_{AB}\partial_M X^A\partial^M X^B\right)\to\sqrt{-g}\left(\frac{1}{2}R_g-\frac{1}{2}\mathrm{e}^{4\phi}\,G_{AB}\,\partial_\mu\hat{X}^A\,\partial^\mu\hat{X}^B\right)\;. \qquad (A.14)
$$

Eqs. (A.13) and (A.14) can be combined into

$$\sqrt{-g}\left(\frac{1}{2}R_g - \frac{1}{2}\,\mathrm{e}^{4\phi}\,G_{AB}\,\partial_\mu z^A\,\partial^\mu \bar{z}^{\bar{B}}\right) \ , \tag{A.15}$$

where z^A is defined as in (2.2). Using (A.11) we compute $g_{A\bar{B}} = \frac{1}{2}\mathrm{e}^{4\phi}\,G_{AB}$, and hence (A.15) can be written as

$$\sqrt{-g}\left(\frac{1}{2}R_g - g_{A\bar{B}}\,\partial_\mu z^A\,\partial^\mu \bar{z}^{\bar{B}}\right) \ . \tag{A.16}$$

In addition, reducing R_G and $G_{AB}F^AF^B$ also gives rise to the four-dimensional gauge kinetic terms

$$\begin{aligned} \sqrt{-G}\,R_G \quad &\rightarrow \quad \sqrt{-g}\left(R_g - \frac{1}{4}\mathrm{e}^{-6\phi}\,F^0\,F^0\right) \ , \\ -\frac{1}{2}\sqrt{-G}\,G_{AB}\,F^A\,F^B \quad &\rightarrow \quad -\frac{1}{2}\sqrt{-g}\,\mathrm{e}^{-2\phi}\,G_{AB}\left[F^AF^B - 2C^B\,F^A\,F^0 + C^A\,C^B\,F^0F^0\right] \ . \end{aligned} \tag{A.17}$$

This we compare with $\mathrm{Im}\mathcal{N}_{IJ}\,F^I\,F^J$ in four dimensions. To this end, we compute the couplings $\mathcal{N}_{IJ}$ for the prepotential (A.7) and we express them in terms of the fields $\hat{X}^A$ and C^A using (2.2),

$$\begin{aligned} \mathcal{N}_{00} &= -\frac{1}{3}\,C_{ABC}\,C^AC^BC^C - \mathrm{i}\left[2\,\mathrm{e}^{-2\phi}\,V\,G_{AB}\,C^AC^B + \hat{V}\right] \ , \\ \mathcal{N}_{0A} &= \frac{1}{2}\,C_{ABC}\,C^BC^C + 2\mathrm{i}\,\mathrm{e}^{-2\phi}\,V\,G_{AB}\,C^B \ , \\ \mathcal{N}_{AB} &= -\,C_{ABC}\,C^C - 2\mathrm{i}\,\mathrm{e}^{-2\phi}\,V\,G_{AB} \ , \end{aligned} \tag{A.18}$$

where

$$\hat{V} = \hat{X}_A\,\hat{X}^A \quad , \quad \hat{X}_A = \frac{1}{6}C_{ABC}\hat{X}^B\hat{X}^C \quad , \quad \mathrm{e}^{-6\phi} = V^{-1}\,\hat{V} \ . \tag{A.19}$$

Hence we find that the sum of the field strength terms on the right hand side of (A.17) equals

$$\frac{1}{4V}\,\mathrm{Im}\mathcal{N}_{IJ}\,F^I\,F^J \ . \tag{A.20}$$

Thus, requiring the matching of the five-dimensional gauge kinetic term $-\frac{1}{4}G_{AB}F^AF^B$ in (A.6) with the four-dimensional gauge kinetic term $\frac{1}{4}\mathrm{Im}\mathcal{N}_{IJ}\,F^I\,F^J$ in (A.10) yields the normalization condition

$$2\,V = 1 \ . \tag{A.21}$$

Next, we reduce the five-dimensional Chern-Simons term $C_{ABC}\,F^A \wedge F^B \wedge A^C$ in (A.6). Using (2.1), we first observe that $C_{ABC}\,F^A \wedge F^B \wedge A^C_\psi\,d\psi$ can be expressed in terms of four-dimensional gauge fields as,

$$\begin{aligned} C_{ABC}\,F^A \wedge F^B \wedge A^C_\psi\,d\psi \quad = \quad & R\,C_{ABC}\,[\,C^A\,F^B \wedge F^C - C^A\,C^B\,F^C \wedge F^0 \\ & + \frac{1}{3}C^A\,C^B\,C^C\,F^0 \wedge F^0\,] \wedge d\psi \ , \end{aligned} \tag{A.22}$$

up to a total derivative term. The field strengths on the right hand side are four-dimensional, and $A^C_\psi = R\, C^C$. Using

$$C_{ABC}\, C^A\, F^B \wedge F^C \wedge d\psi = -\frac{1}{2} d\psi\, d^4x\, \sqrt{-g}\, C_{ABC}\, C^A\, F^B\, \tilde{F}^C \ , \tag{A.23}$$

and similarly for the other terms in (A.22), we obtain (up to a total derivative)

$$C_{ABC}\, F^A \wedge F^B \wedge A^C_\psi\, d\psi = \frac{1}{2}\, R\, d\psi\, d^4x\, \sqrt{-g}\, \mathrm{Re}\mathcal{N}_{IJ}\, F^I\, \tilde{F}^J \ , \tag{A.24}$$

where we used (A.18). Then, using

$$C_{ABC}\, F^A \wedge F^B \wedge A^C = 3 C_{ABC}\, F^A \wedge F^B \wedge A^C_\psi\, d\psi \ , \tag{A.25}$$

which holds up to a total derivative term, we obtain

$$\frac{1}{6G_5} \int C_{ABC}\, F^A \wedge F^B \wedge A^C = \frac{1}{4G_4} \int d^4x\, \sqrt{-g}\, \mathrm{Re}\mathcal{N}_{IJ}\, F^I\, \tilde{F}^J \ . \tag{A.26}$$

Thus, dimensional reduction of (A.6) yields (A.10), up to boundary terms.

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